Calculer $\displaystyle\int \frac{dx}{1+\sin^2 x}$.

En posant $t = \tan x$, on a $dt = \dfrac{dx}{\cos^2 x} = (1+\tan^2 x)\ dx = (1+t^2)\ dx$ et donc $dx = \dfrac{dt}{1+t^2}$. De même, $t^2 = \tan^2 x = \dfrac{\sin^2 x}{\cos^2 x} = \dfrac{\sin^2 x}{1-\sin^2 x}$ d'où on déduit que $\sin^2 x = \dfrac{t^2}{1+t^2}$.
On a

$$\begin{aligned}
\int \frac{dx}{1+\sin^2 x} &= \int \frac{1}{1+\dfrac{t^2}{1+t^2}} \cdot \frac{dt}{1+t^2} \\
&= \int \frac{dt}{2t^2+1} \\
&= \frac{1}{2}\int \frac{dt}{t^2+\dfrac{1}{2}} \\
&= \frac{1}{2}\cdot\frac{1}{\sqrt{\frac{1}{2}}}\arctan\frac{t}{\sqrt{\frac{1}{2}}} + K \\
&= \boxed{\sqrt{2}\arctan\left(\sqrt{2}\tan x\right) + K \qquad \text{avec } K \in \mathbb{R}}
\end{aligned}$$