

Calculer  $\int \frac{dx}{2 + \cos^2 x}$ .

En posant  $t = \tan x$ , on a  $dt = \frac{dx}{\cos^2 x} = (1 + \tan^2 x) dx = (1 + t^2) dx$  et donc  $dx = \frac{dt}{1 + t^2}$ . De plus,  $\cos^2 x = \frac{1}{1 + \tan^2 x} = \frac{1}{1 + t^2}$ .

On a

$$\int \frac{dx}{2 + \cos^2 x} = \int \frac{1}{2 + \frac{1}{1 + t^2}} \cdot \frac{dt}{1 + t^2}$$

$$= \int \frac{dt}{2t^2 + 3}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + \frac{3}{2}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{3}{2}}} \arctan \frac{t}{\sqrt{\frac{3}{2}}} + K$$

$$= \frac{\sqrt{6}}{6} \arctan \left( \sqrt{\frac{2}{3}} \tan x \right) + K \quad \text{avec } K \in \mathbb{R}$$