Calculer $I = \displaystyle\int \frac{4+\sin x}{\sin x.(1+\cos x)}\ dx$.

En utilisant le changement de variable $u = \tan\dfrac{x}{2}$ (c'est-à-dire $x = 2\arctan u$), on a $dx = \dfrac{2\ du}{1+u^2}$, mais aussi $\sin x = \dfrac{2u}{1+u^2}$ et $\cos x = \dfrac{1-u^2}{1+u^2}$.

On a alors

$$\begin{aligned}
\int \frac{4+\sin x}{\sin x.(1+\cos x)}\ dx &= \int \frac{4+\dfrac{2u}{1+u^2}}{\dfrac{2u}{1+u^2}\cdot\left(1+\dfrac{1-u^2}{1+u^2}\right)}\ \frac{2\ du}{1+u^2} \\
&= \int \frac{4u^2+2u+4}{2u}\ du \\
&= \int \left(2u+1+\frac{2}{u}\right)\ du \\
&= u^2+u+2\ln|u|+K \\
&= \boxed{\tan^2\frac{x}{2}+\tan\frac{x}{2}+2\ln\left|\tan\frac{x}{2}\right|+K \text{ avec } K\in\mathbb{R}}
\end{aligned}$$