Calculer, en commençant par le changement de variable $t = \sqrt{e^x - 1}$:

$$\int_{\ln 2}^{2\ln 2} \frac{3}{(2+e^x)\sqrt{e^x-1}}\,dx$$

En posant $t = \sqrt{e^x - 1}$, on a $t^2 = e^x - 1$ et donc $2t\;dt = e^x\;dx$. L'intégrale de l'énoncé se transforme donc en

$$\begin{aligned}
\int_{\ln 2}^{2\ln 2} \frac{3}{(2+e^x)\sqrt{e^x-1}}\,dx &= \int_{\ln 2}^{2\ln 2} \frac{3e^x}{e^x(2+e^x)\sqrt{e^x-1}}\,dx \\
&= \int_{1}^{\sqrt{3}} \frac{6t}{(t^2+1)\cdot(t^2+3)\cdot t}\,dt \\
&= \int_{1}^{\sqrt{3}} \frac{6}{(t^2+1)\cdot(t^2+3)}\,dt
\end{aligned}$$

Décomposons cette intégrale en deux fractions simples.

$$\begin{aligned}
\int_{1}^{\sqrt{3}} \frac{6}{(t^2+1)\cdot(t^2+3)}\,dt &= \int_{1}^{\sqrt{3}} \left(\frac{At+B}{t^2+1} + \frac{Ct+D}{t^2+3}\right)\,dt \\
&= \int_{1}^{\sqrt{3}} \frac{(At+B)(t^2+3)+(Ct+D)(t^2+1)}{(t^2+1)(t^2+3)}\,dt \\
&= \int_{1}^{\sqrt{3}} \frac{(A+C)t^3+(B+D)t^2+(3A+C)t+(3B+D)}{(t^2+1)(t^2+3)}\,dt
\end{aligned}$$

On en tire

$$\begin{cases} A+C=0 \\ B+D=0 \\ 3A+C=0 \\ 3B+D=6 \end{cases} \iff \begin{cases} A=0 \\ B=3 \\ C=0 \\ D=-3 \end{cases}$$

$$\begin{aligned}
\int_{1}^{\sqrt{3}} \frac{6}{(t^2+1)\cdot(t^2+3)}\,dt &= \int_{1}^{\sqrt{3}} \left(\frac{3}{t^2+1} - \frac{3}{t^2+3}\right)\,dt \\
&= 3\left(\arctan t - \frac{1}{\sqrt{3}}\arctan\frac{t}{\sqrt{3}}\right]_1^{\sqrt{3}}\Bigg) \\
&= 3\left(\left(\arctan\sqrt{3} - \frac{1}{\sqrt{3}}\arctan 1\right) - \left(\arctan 1 - \frac{1}{\sqrt{3}}\arctan\frac{\sqrt{3}}{3}\right)\right) \\
&= 3\left(\left(\frac{\pi}{3} - \frac{1}{\sqrt{3}}\cdot\frac{\pi}{4}\right) - \left(\frac{\pi}{4} - \frac{1}{\sqrt{3}}\cdot\frac{\pi}{6}\right)\right) \\
&= \boxed{\frac{\pi}{12}\cdot(3-\sqrt{3})}
\end{aligned}$$