

Calculer $I = \int \frac{\cos 4x}{(\cot x - \tan x)^2} dx.$

En utilisant différentes formules de trigonométrie, on a

$$\begin{aligned}
 I &= \int \frac{\cos 4x}{(\cot x - \tan x)^2} dx \\
 &= \int \frac{\cos 4x}{\left(\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\right)^2} dx \\
 &= \int \frac{\cos 4x}{\left(\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}\right)^2} dx \\
 &= \frac{1}{4} \int \frac{\cos 4x \cdot \sin^2 2x}{\cos^2 2x} dx \\
 &= \frac{1}{4} \int \frac{(2\cos^2 2x - 1) \cdot \sin^2 2x}{\cos^2 2x} dx \\
 &= \frac{1}{4} \int (2\sin^2 2x - \tan^2 2x) dx \\
 &= \frac{1}{4} \int \left(1 - \cos 4x + 1 - \frac{1}{\cos^2 2x}\right) dx \\
 &= \frac{1}{4} \left(2x - \frac{1}{4} \sin 4x - \frac{1}{2} \tan 2x\right) + K \\
 &= \boxed{\frac{x}{2} - \frac{1}{16} \sin 4x - \frac{1}{8} \tan 2x + K}
 \end{aligned}$$