Calculer, en utilisant la règle de l'Hospital,

$$L = \lim_{x \to +\infty} x^2 \left(e^{1/x} - e^{1/(x+1)} \right)$$

Posons $t = \dfrac{1}{x}$ (et donc $x = \dfrac{1}{t}$ et $x + 1 = \dfrac{1+t}{t}$). Si $x \to +\infty$, alors $t \to 0^+$.
On a successivement :

$$\begin{aligned}
L &= \lim_{x \to +\infty} x^2 \left(e^{1/x} - e^{1/(x+1)} \right) \\
&= \lim_{t \to 0^+} \frac{e^y - e^{y/(1+y)}}{y^2} \\
&= \left[\frac{0}{0} \right] \\
&\overset{(H)}{=} \lim_{t \to 0^+} \frac{e^y - e^{y/(1+y)} \cdot \dfrac{1}{(1+y)^2}}{2y} \\
&= \left[\frac{0}{0} \right] \\
&\overset{(H)}{=} \lim_{t \to 0^+} \frac{e^y - e^{y/(1+y)} \cdot \dfrac{1}{(1+y)^4} + e^{y/(1+y)} \cdot \dfrac{2}{(1+y)^3}}{2} \\
&= \frac{1 - 1 + 2}{2} \\
&= \boxed{1}
\end{aligned}$$