Calculer $\displaystyle\int \frac{dx}{\lambda+\cos x}$ en discutant en fonction de $\lambda \in \mathbb{R}$.

Posons classiquement $t = \tan\frac{x}{2}$, et donc $dx = \dfrac{2\ dt}{1+t^2}$, avec $\cos x = \dfrac{1-t^2}{1+t^2}$.
Ainsi,

$$\int \frac{dx}{\lambda+\cos x} = \int \frac{2}{(1+t^2)\left(\lambda + \dfrac{1-t^2}{1+t^2}\right)}\,dt = \int \frac{2}{(\lambda-1)t^2+(\lambda+1)}\,dt$$

- Si $\lambda = 1$, alors $I = \displaystyle\int \frac{2\ dt}{2} = t + K = \boxed{\tan\frac{x}{2} + K}$.
- Si $\lambda = -1$, alors $I = \displaystyle\int \frac{2\ dt}{-2t^2} = \frac{1}{t} + K = \boxed{\cot\frac{x}{2} + K}$.
- Si $\lambda \neq \pm 1$, alors $I = \displaystyle\frac{2}{\lambda-1}\int \frac{dt}{\dfrac{\lambda+1}{\lambda-1}+t^2} = \frac{2}{\lambda-1}\int\frac{dt}{A+t^2}$, avec $A = \dfrac{\lambda+1}{\lambda-1}$.
  - si $A > 0$ (c'est-à-dire si $\lambda < -1$ ou $\lambda > 1$), alors

$$\begin{aligned} I &= \frac{2}{\lambda-1}\frac{1}{\sqrt{A}}\arctan\frac{t}{\sqrt{A}} + K \\ &= \boxed{\frac{2}{(\lambda-1)\sqrt{A}}\arctan\frac{\tan\frac{x}{2}}{\sqrt{A}} + K} \end{aligned}$$

  - si $A < 0$ (c'est-à-dire si $-1 < \lambda < 1$), alors

$$\begin{aligned} I &= \frac{2}{\lambda-1}\int\frac{dt}{t^2-(-A)} \\ &= \frac{1}{(\lambda-1)\sqrt{-A}}\ \ln\left|\frac{t-\sqrt{-A}}{t+\sqrt{-A}}\right| + K \\ &= \boxed{\frac{1}{(\lambda-1)\sqrt{-A}}\ \ln\left|\frac{\tan\frac{x}{2}-\sqrt{-A}}{\tan\frac{x}{2}+\sqrt{-A}}\right| + K} \end{aligned}$$